Problems: Flux Through Surfaces

Let $\mathbf{F} = (x, y, z)$.

1. Find the flux of $\mathbf{F}$ through the square with vertices $(0,0,0)$, $(1,0,0)$, $(1,1,0)$, $(0,1,0)$.

   **Answer:** The square in question lies in the plane $z = 0$, so $\mathbf{n} = (0,0,1)$. $\mathbf{F} \cdot \mathbf{n} = z = 0$ on the whole square, so the flux is zero.

2. Find the flux of $\mathbf{F}$ through the square with vertices $(0,0,1)$, $(1,0,1)$, $(1,1,1)$, $(0,1,1)$.

   **Answer:** Again $\mathbf{n} = (0,0,1)$ and $\mathbf{F} \cdot \mathbf{n} = z$.

   $$\text{Flux} = \iint_S \mathbf{F} \cdot \mathbf{n} \, dS = \int_0^1 \int_0^1 1 \, dx \, dy = 1.$$ 

3. Find the flux of $\mathbf{F}$ through the surface $x^2 + y^2 = 1$ with $0 \leq z \leq 1$.

   **Answer:** Here $\mathbf{n} = (x, y, 0)$, so $\mathbf{F} \cdot \mathbf{n} = x^2 + y^2 = 1$. We can parametrize the surface by $x = \cos \theta$, $y = \sin \theta$ with $dS = d\theta \, dz$ and integrate, or we can observe that the result of that calculation will just be the surface area of the cylinder. Flux = $2\pi$. 