Problems: Surface Independence

Suppose that $\mathbf{F} = \nabla \times \mathbf{G}$, where the components of $\mathbf{G}$ have continuous second partial derivatives. Suppose also that $S$ is a closed, positively-oriented surface divided into two parts by a closed curve $C$. Apply Stokes' theorem to show that $\iint_S \mathbf{F} \cdot \mathbf{n} dS = 0$.

**Answer:** We start by drawing a picture.

$$
\begin{array}{c}
\text{By Stokes' theorem, } \oint_C \mathbf{G} \cdot d\mathbf{r} = \iint_{S_1} \text{curl} \mathbf{G} \, dA = \iint_{S_1} \mathbf{F} \cdot \mathbf{n} dS. \text{ Similarly,}
\end{array}
$$

$$
\begin{array}{c}
\oint_{-C} \mathbf{G} \cdot d\mathbf{r} = \iint_{S_2} \mathbf{F} \cdot \mathbf{n} dS.
\end{array}
$$

We sum the two results to get:

$$
\iint_S \mathbf{F} \cdot \mathbf{n} dS = \iint_{S_1} \mathbf{F} \cdot \mathbf{n} dS + \iint_{S_2} \mathbf{F} \cdot \mathbf{n} dS = \oint_C \mathbf{G} \cdot d\mathbf{r} + \oint_{-C} \mathbf{G} \cdot d\mathbf{r} = 0.
$$
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