1. Began to discuss nonlinear systems and the relation to linear systems, e.g. for a pendulum, \( \ddot{\theta} = \omega^2 \sin(\theta) \), but for \( \theta \) and \( t \) small, approximately the same as \( \ddot{\Theta} = \omega^2 \dot{\theta} \).

2. Defined “structurally stable” (I’m not certain what the conventional term is). Given a nonlinear system \( x' = F(x,t) \) (or just \( = F(x)\) for an autonomous system), a property of the system is structurally stable if for every continuous \( G(x,t) \) (or just \( G(x) \) for an autonomous system), there exists \( \varepsilon_0 = \varepsilon_0(F,G) \) such that the property holds for \( x' = F + \varepsilon G \), \( |\varepsilon| < \varepsilon_0 \).

Gave the example of the number of equilibrium points.

\textbf{Prop:} Let \( R \subset \mathbb{R}^n \) be a bounded closed region. If

1. these are no equilibrium points or \( \partial R \)
2. these are only finitely many equilibrium points in \( \text{Int}(R) \),
3. every equilibrium point is nondegenerate, i.e. \( \left| \frac{\partial F_i}{\partial R_j} \right| \neq 0 \) at the equilibrium point, then \( \exists \varepsilon_0 > 0 \) such that for all \( -\varepsilon_0 < \varepsilon < \varepsilon_0 \), (1), (2) + (3) hold for \( F + \varepsilon G \). Moreover the number of equilibrium points is constant.

\textbf{Pf:} Use the implicit function theorem for \( F + \varepsilon G = R \times \mathbb{R} \rightarrow \mathbb{W} \)

3. Defined orbits + orbital portrait for a system (obviously closely related to the orbital portraits from Ch.3)

Defined nullclines. Worked through the example

\[
\begin{align*}
x' &= xy \\
y' &= x^2 - y^2
\end{align*}
\]

Guess these are solutions \( y = mx \) and solve to get \( m^2 = \frac{1}{2} \)

Orbits look roughly like the contour curves of a monkey saddle.
4. Gave algorithm for sketching an orbital portrait for an autonomous 2D system.

**Step 1**: Find all equilibrium points.

**Step 2**: For each equilibrium point, draw the “local picture” (if it is nondegenerate + structurally stable)

**Step 3**: Draw the nullclines and other “fences” (i.e. curves that help to determine basins of attraction).

**Step 4**: Interpolate between the local pictures to give a rough sketch.

Went through the steps for \[
\begin{align*}
x' &= x(y - 1) \\
y' &= y(x - 1)
\end{align*}
\]