18.034 Problem Set #3

Due by Friday, March 6, 2009, by NOON.

1. This problem pertains to the differential equation $y'' + \omega^2 y = \sin \omega_0 t$, where $\omega \neq 0$ and $\omega_0$ is close to but different from $\omega$. (c) Check that $y_2(t) = \frac{\sin \omega_0 t - \sin \omega t}{\omega^2 - \omega_0^2}$ is the particular solution for which the initial conditions remain finite as $\omega_0 \to \omega$.

2. Let $f(x)$ and $g(x)$ be two solutions of the differential equation $y' = F(x,y)$ in a domain where $F$ satisfies the condition*: 
\[ y_1 < y_2 \quad \text{implies} \quad F(x,y_2) - F(x,y_1) \leq L(y_2 - y_1). \]

3. Very that $(\sin x)/x$, $x$ satisfy the following equations, respectively, and thus obtain the second solution.
   (a) $xy'' + 2y' + xy = 0 \quad (x > 0),$
   (b) $(2x - 1)y'' - 4xy' + 4y = 0 \quad (2x > 1)$.

4. (a) Birkhoff-Rota, pp. 57, #4. (Typo. $I(x) = q - p^2/4 - p'/2$.)
   (b) Birkhoff-Rota, pp. 57, #7(a). (Use part (a) instead of #6 as is suggested in the text.)
   (c) Birkhoff-Rota, pp. 57, #7(b).

5. Let $(\cosh x)y'' + (\cos x)y' = (1 + x^2)y$ for $a < x < b$ and let $y(a) = y(b) = 1$. Show that $0 < y(x) < 1$ for $a < x < b$.

6. (a) Birkhoff-Rota, pp. 75, #3, (b) Birkhoff-Rota, pp. 75, #4.

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*It is called a one-sided Lipschitz condition.