18.034 Honors Differential Equations
Spring 2009

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18.034 Practice Midterm #1

1. Solve the initial value problem
   \[ y'' - 3y^2 = 0, \quad y(0) = 2, \quad y'(0) = 4. \]
   Determine the interval in which the solution exists.

2. Consider the differential equation \( y' = (1 - y)(y - 2)^3 \).
   (a) Sketch the graph of \( f(y) = (1 - y)(y - 2)^3 \).
   (b) Determine the critical points (stationary solutions).
   (c) Discuss the stability of critical points in part (b).

3. Determine the values of \( a \), if any, for which all solutions of the differential equation
   \[ y'' - (2a - 1)y' + a(a - 1)y = 0 \]
   tend to zero as \( t \to \infty \). Also, determine the values of \( a \), if any, for which all (nonzero) solutions become unbounded as \( t \to \infty \).

4. Consider the undamped forced vibration system
   \[ y'' + \omega^2 y = \sin 2t, \quad u(0) = 0, \quad u'(0) = 0. \]
   (a) Find the solution for \( \omega = 2 \).
   (b) Find the solution for \( \omega = 2 \).

5. (a) Find the value \( m \) for which \( y = t^m \) is a solution of the differential equation
   \[ t^2y'' - 13ty' + 49y = 0, \quad t > 0. \]
   (b) Find a second solution of the differential equation in part (a).

6. Show that every solution of \( u'' + (1 + e^t)u = 0 \) vanishes infinitely often on \( 0 < t < \infty \).