1. (a) Find numbers $a$ and $b$ so that the differential equation $t^2y'' + aty' + by = 0$ has solutions $t^2$ and $t^3$ on the interval $t \in (0, \infty)$.
(b) Find a differential equation that has solutions $(1 - t)^2$ and $(1 - t)^3$ on the interval $t \in (-\infty, 1)$.
(c) Find a differential equation that has solutions $t$ and $e^t$.

2. Using variation of parameters find a solution of $y'' - \left(\frac{2}{t^2}\right)y = t, t \neq 0$.

3. Find a general solution of $(D^2 - 1)^4(D^3 + 1)^5y = 3e^t$.

4. Show that the function $u = e^z$ is a solution of $y'' + p(t)y' + q(t)y = 0$ if and only if $z$ is a solution of the Riccati equation $y' + p(t)y + q(t) = -y^2$.

5. (a) State the existence and uniqueness theorem for the initial value problem
\[ y' = f(t, y), \quad y(t_0) = y_0. \]
(b) Show that $f(t, y) = -y + 1$ satisfies the Lipschitz condition for all $t$ and $y$.
(c) Using Picard’s iteration method obtain the iterate $y_1(t)$ and $y_2(t)$ of
\[ y' = -y + 1, \quad y(0) = 1. \]
(d) Find the exact solution of the initial value problem in part (c).