1. Under what conditions on $b$ and $k$ do all solutions $y(t)$ to

$$y'' + by' + ky = 0$$

tend to zero as $t \to \infty$? What is the physical significance of these conditions for a spring system?

2. Let $u$ and $v$ be continuous and linearly independent on an interval $I$. Suppose $w$ is a function on $I$ with only finitely many zeros.

   (a) Show that $wu$ and $wv$ are linearly independent on $I$.
   
   (b) You can’t use the Wronskian in this problem. Why not?
   
   (c) Show that the result can fail if $u$ and $v$ are not continuous.

3. Show that $e^t$, $e^{-t}$, and $e^{2t}$ are linearly independent on $\mathbb{R}$ without using the Wronskian.

4. Show that a function $y$ satisfying

$$e^x y'' + (\sin x)y' - (1 + x)y \geq 0, \quad y(0) \geq 0, y'(0) > 0,$$

must be strictly increasing.

5. Consider the problem

$$w''' + \lambda qw = 0, \quad w(a) = w(b) = 0$$

where $\lambda \in \mathbb{R}$ and $q = q(x)$ is a positive function of $x$. Show that there are no non-trivial solutions if $\lambda < 0$. 