18.034 Honors Differential Equations
Spring 2009

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1. (Birkhoff-Rota, #2,3, p. 75) Find a basis of real solutions to the ODE

   (a) $u''' - u,$

   (b) $u^{(IV)} = u.$

2. (Birkhoff-Rota, #2, p.82)

   Find a constant coefficient linear operator $L$ such that $e^{-t}$, $te^{-t}$, and $e^t$
   are a basis of solutions for the ODE $L[u] = 0$. Then find bases for the
   second- and third-order ODE $L^2[u] = 0$ and $L^3[u] = 0$.

3. (Birkhoff-Rota, #4, p. 82)

   Knowing bases of solutions for $L_1[u] = 0$ and $L_2[u] = 0$, find a basis of
   solutions for $(L_1 \circ L_2)[u] = 0$.

4. (Birkhoff-Rota, #5, p. 82)

   Show that every linear differential operator $L$ with constant real coefficients
   can be factored as $L = AL_1 \circ L_2 \circ \cdots \circ L_m$ where $A \in \mathbb{R}$ and
   $L_i = D_i + b_i$ or $L_i = D^2 + p_i D + q_i$.

5. (Birkhoff-Rota, #8, p. 82)

   Prove that $u'' + 2iu' + 3u = 0$ has no non-trivial real solution.