18.034 Recitation: March 10th, 2009

1. Let $I \subset \mathbb{R}$ be an interval and recall that a function $f : I \to \mathbb{R}$ is said to be \textit{Lipschitz} on $I$ if there is a constant $C$ such that $|f(x) - f(y)| \leq C|x - y|$ for all $x, y \in I$.

   (a) Show that if $f$ is Lipschitz on $I$, then $f \in C(I)$.

   (b) Show that if $f$ is differentiable on $I$ with bounded derivative, then $f$ is Lipschitz on $I$.

   (c) Show that $f(x) = |x|$ is Lipschitz on $\mathbb{R}$.

   (d) Show that $f(x) = e^x$ is not Lipschitz on $\mathbb{R}$.

2. (Birkhoff-Rota, p. 62, #3)

   Solve
   
   $$y'' + 3y' + 2y = x^3$$

   for the initial conditions $y'(0) = y(0) = 0$.

3. (Birkhoff-Rota, p. 62, #4)

   Show that any second-order linear inhomogeneous equation that has both $x^2$ and $\sin^2 x$ as solutions must have a singular point at the origin.

4. Describe the dominant behavior as $t \to \infty$ of any solution $u$ to

   $$(D + 3)(D^2 + 1)^5 u = 640 \cos t.$$

   \textit{Hint: the function}

   $$U_0(t) = \frac{t^5}{5!} (6 \sin t - 2 \cos t)$$

   \textit{solves the equation.}