1. (a) In a perfect environment, the population of Norway rat that breeds on the MIT campus increases by a factor of $e \approx 2.718281828459045 \ldots$ each year. Model this natural growth by a differential equation. What is the growth rate $k$?

(b) MIT is a limited environment, with a maximal sustainable Norway rat population of $R = 1000$ rats. Write down the logistic equation modeling this. (You may use "$k$" for the natural growth rate here if you failed to find it in (a).)

(c) The MIT pest control service intends to control these rats by killing them at a constant rate of $a$ rats per year. If it wants to limit the rat population to 75% of the maximal sustainable population, what rate $a$ it should aim for (in rats per year)?
2. For the autonomous equation \( \dot{x} = x(x - 1)(x + 2) \), please sketch:

(a) the phase line, identifying the critical points and whether they are stable, unstable, or neither.

(b) at least one solution of each basic type (so that every solution is a time-translate of one you have drawn)

Below is a diagram of a direction field of the differential equation \( y' = (1/4)(x - y^2) \). On it please plot and label:

(c) the nullcline

(d) at least two quite different solutions

(e) the separatrix (if there is one)

(f) True or false: If \( y(x) \) is a solution with a minimum, then for all large enough \( x \), \( y(x) < \sqrt{x} \). (No explanation needed: just circle one.)
3. **(a)** Use Euler’s method with stepsize $h = 1/2$ to estimate the value at $x = 3/2$ of the solution to $y' = x + y$ such $y(0) = 1$. 

**(b)** Find the solution of $t\dot{x} + x = \cos t$ such that $x(\pi) = 1$. 


4. (a) Find real $a, b$ such that \( \frac{1}{3+2i} = a + bi \). \[3\]

(b) Find real $r, \theta$ such that $1 - i = re^{i\theta}$. \[3\]

(c) Find real $a, b$ such that $(1 - i)^8 = a + bi$. \[3\]

(d) Find real $a, b$ such that $b > 0$ and $a + bi$ is a cube root of $-1$. \[3\]

(e) Find real $a, b$ such that $e^{\ln 2 + i\pi} = a + bi$. \[3\]

(f) Write $f(t) = 2\cos(4t) - 2\sin(4t)$ in the form $A\cos(\omega t - \phi)$. \[5\]
5. (a) Find a particular solution to the equation $\dot{x} + 3x = e^{2t}$. [5]

(b) Find the solution to the same equation such that $x(0) = 1$. [5]

(c) Write down a linear equation with exponential right hand side of which $\dot{x} + 3x = \cos(2t)$ is the real part. [5]

(d) Find a particular solution to the equation $\dot{x} + 3x = \cos(2t)$. [5]
18.03SC Differential Equations
Fall 2011

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.