Part II Problems

Problem 1:  [Sinusoidal input and output]

(a) Express \( \text{Re} \left( \frac{e^{3it}}{\sqrt{3} + i} \right) \) in the form \( a \cos(3t) + b \sin(3t) \). Then rewrite this in the form \( A \cos(3t - \phi) \). Now find this same answer using the following method. By finding its modulus and argument, write \( \sqrt{3} + i \) in the form \( Ae^{i\phi} \). Then substitute this into \( \frac{e^{3it}}{\sqrt{3} + i} \), and use properties of the exponential function to find \( B \) and \( \phi \) such that \( \frac{e^{3it}}{\sqrt{3} + i} = Be^{i(3t-\phi)} \). Finally, take the real part of this new expression.

(b) Find a solution to the differential equation \( \dot{z} + 3z = e^{2it} \) of the form \( we^{2it} \), where \( w \) is some complex number. What is the general solution?

(c) Find a solution of \( \dot{x} + 3x = \cos(2t) \) by relating this ODE to the one in (b). What is the general solution?