Inflection Points

Quiz: Inflection Points.
When do nonconstant solutions of the autonomous ODE $\dot{y} = f(y)$ have inflection points.

Choices:

a) when $f(y) = 0$
b) when $f'(y) = 0$
c) when $f''(y) = 0$

Answer: (b) $f'(y) = 0$.
By the chain rule

$$\frac{dy}{dt} = f(y) \Rightarrow \frac{d^2y}{dt^2} = f'(y) \frac{dy}{dt}.$$ 

An inflection point is one where $\frac{d^2y}{dt^2} = 0$. By the above formula this occurs when either $f'(y) = 0$ or $\dot{y} = 0$. Since $\dot{y} = 0$ only on constant solutions, which have no inflection points, all that’s left is $f'(y) = 0$. 
