Part I Problems and Solutions

For each of the following ODE’s, draw a direction field by using about five isoclines; the picture should be square, using the intervals between -2 and 2 on both axes. Then sketch in some integral curves, using the information provided by the direction field. Finally, do whatever else is asked.

Problem 1: \( y' = -\frac{y}{x} \). Solve the equation exactly and compare your integral curves with the correct ones.

Solution: \( y' = -\frac{y}{x} \). Isoclines: \( -\frac{y}{x} = m \rightarrow y = -mx \).
Solutions: \( \frac{dy}{y} = -\frac{dx}{x} \rightarrow \ln y = -\ln x + c \rightarrow y = \frac{c}{x} \).

Problem 2: \( y' = 2x + y \). Find a solution whose graph is also an isocline, and verify this fact analytically (i.e., by calculation, and not from a picture).

Solution: \( y' = 2x + y \) has isoclines \( 2x + y = m \rightarrow y = -2x + m \). Isocline \( y = -2x + m \) is also a solution if \( y' = -2 \) (from solution) and also \( y' = 2x + y \) (from DE) \( \rightarrow y = -2x - 2 \), that is, the isocline with \( m = -2 \).

Problem 3: \( y' = \frac{1}{x+y} \). Use the interval -3 to 3 on both axes; draw in the integral curves that pass respectively through (0,0), (-1,1), (0,-2). Will these curves cross the line \( y = -x - 1 \)? Explain by using the Intersection Principle.
**Solution:** Isoclines \( x + y = \frac{1}{m} \rightarrow y = -x + \frac{1}{m} \).

\( y = -x - 1 \) is an integral curve (or solution) so other solutions cannot cross it.
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