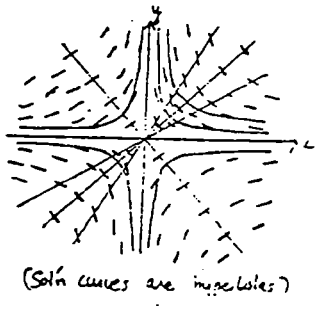
## Part I Problems and Solutions

*For each of the following ODE's, draw a direction field by using about five isoclines; the picture should be square, using the intervals between -2 and 2 on both axes. Then sketch in some integral curves, using the information provided by the direction field. Finally, do whatever else is asked.*

**Problem 1:** $y' = -\frac{y}{x}$. *Solve the equation exactly and compare your integral curves with the correct ones.*

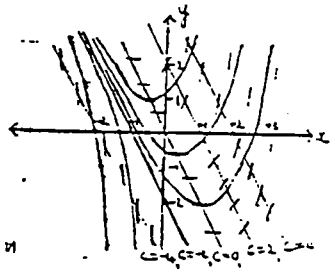
**Solution:** $y' = -\frac{y}{x}$. Isoclines: $-\frac{y}{x} = m \to y = -mx$.

Solutions: $\frac{dy}{y} = -\frac{dx}{x} \to \ln y = -\ln x + c \to y = \frac{c}{x}$.




**Problem 2:** $y' = 2x + y$. *Find a solution whose graph is also an isocline, and verify this fact analytically (i.e., by calculation, and not from a picture).*

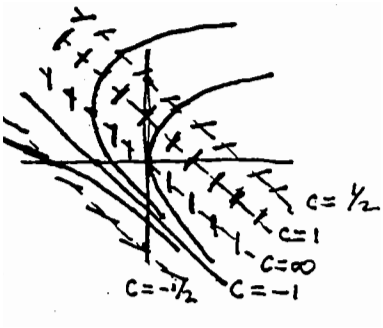
**Solution:** $y' = 2x + y$ has isoclines $2x + y = m \to y = -2x + m$. Isocline $y = -2x + m$ is also a solution if $y' = -2$ (from solution) and also $y' = 2x + y$ (from DE) $\to y = -2x - 2$, that is, the isocline with $m = -2$.



**Problem 3:** $y' = \frac{1}{x+y}$. *Use the interval $-3$ to 3 on both axes; draw in the integral curves that pass respectively through $(0,0)$, $(-1,1)$, $(0,-2)$. Will these curves cross the line $y = -x - 1$? Explain by using the Intersection Principle.*

**Solution:** Isoclines $x + y = \frac{1}{m} \to y = -x + \frac{1}{m}$.

$y = -x - 1$ is an integral curve (or solution) so other solutions cannot cross it.



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