Part I Problems and Solutions

Problem 1: A driven spring-mass-dashpot system is modeled by the DE

\[ m \ddot{x} + c \dot{x} + kx = F_0 \cos \omega t \]

with \( m = 1 \), \( c = 6 \), and \( k = 45 \). \( F_0 = 50 \). Find the amplitude \( A(\omega) \) of the response as a function of the input frequency \( \omega \) and find the frequency which gives the largest system response. Is this a system for which ‘practical resonance’ occurs?

Solution: Using the formulas derived in this session, we have

\[ A(\omega) = F_0 \left( (k - m\omega^2)^2 + c^2 \omega^2 \right)^{\frac{1}{2}} \]

\[ A(\omega) = 50 \left( (45 - \omega^2)^2 + 36 \omega^2 \right)^{\frac{1}{2}} \]

\[ \omega_{\text{max}} = \left( \frac{k}{m} - \frac{1}{2} \left( \frac{c}{m} \right)^2 \right)^{\frac{1}{2}} = \text{the frequency which gives practical resonance if} \]

\( c < \sqrt{4km} \). In this case, \( c = 6 < \sqrt{4 \cdot 45 \cdot 1} = 6\sqrt{5} \). So the maximum system response occurs when \( \omega_{\text{max}} = \sqrt{\frac{45}{1} - \frac{136}{2}} = \sqrt{27} \approx 5.196 (\text{rad sec}) \).