Part I Problems and Solutions

Problem 1:  a) Find the periodic solution to

\[ x''' + x = 2 \cos t \]

in amplitude-phase form.

b) What is the gain and the phase lag?

Solution:  a) characteristic polynomial \( p(s) = s^3 + 1 \);
complex replacement: \( z''' + z = 2e^{it} \), \( x = \text{Re}(z) \)

ERF: \( p(i) = i^3 + 1 = -i + 1 \);

\[ z_p = \frac{2}{p(i)} e^{it} \]

\[ \frac{1}{p(i)} = \frac{1}{1-i} = \frac{1}{\sqrt{2}} e^{\frac{i\pi}{4}} \]

\[ z_p = \frac{2}{\sqrt{2}} e^{\frac{i\pi}{4}} \Rightarrow x_p = \sqrt{2} \cos(t + \frac{\pi}{4}) \]

b) Gain = \( \left| \frac{1}{p(i)} \right| = \frac{1}{\sqrt{2}} \);  
Phase lag \( \phi = -\frac{\pi}{4} \)