Introduction

In this session we will continue to develop the important case of linear constant coefficient DE’s with sinusodial input.

We will start by defining stability. In a stable system the response to a periodic input will be essentially periodic. The word essentially indicates that there will be some transient behavior depending on the initial conditions, but this will die away over time.

For constant coefficient equations the important fact is that stability is equivalent to all the characteristic roots being negative, or if they are complex having negative real part. This will turn out to be a simple consequence of the fact the if $a$ is negative then $e^{at}$ goes to 0 as $t$ grows to infinity.

The other main goal of this session is to introduce the operator $D$ and the notation $p(D)$. We will use this to rephrase constant coefficient differential equations and to write elegant formulas for the gain, phase lag and response to sinusoidal input.