Part I Problems and Solutions

Problem 1: Find a particular solution to the DE

\[ x''' + x' = 2 \cos t \]

Solution: Characteristic polynomial:  \( p(s) = s^3 + s \);
Complex replacement:  \( z''' + z' = 2e^{it} \),  \( x = \text{Re}(z) \)
Resonant Response formula:  \( p(i) = i^3 + i = -i + i = 0; \)
\( p'(s) = 3s^2 + 1, \ p'(i) = 3i^2 + 1 = -3 + 1 = -2 \)
\( z_p = \frac{2}{p'(i)} t e^{it} = -t e^{it} \)
\( x_p = \text{Re}(z_p) = -t \cos t. \)