Part I Problems and Solutions

Problem 1:
Find the solution satisfying the initial conditions:
\[ y'' - y = x^2, \quad y(0) = 0, \quad y'(0) = -1 \]

Solution: \( y_h = c_1 e^x + c_2 e^{-x} \).

Try:
\[
\begin{align*}
y_p &= a_1 x^2 + a_2 x + a_3 \\
y''_p &= 2a_1 \\
x^2 &= -a_1 x^2 - a_2 x + 2a_1 - a_3
\end{align*}
\]

Thus \( a_1 = -1, a_2 = 0, a_3 = -2 \). So
\[
y = c_1 e^x + c_2 e^{-x} - x^2 - 2
\]

\( y(0) = 0 \) gives us \( c_1 + c_2 - 2 = 0 \); \( y'(0) = -1 \) gives us \( c_1 - c_2 = -1 \), so \( c_1 = \frac{1}{2}, c_2 = \frac{3}{2} \).

Thus,
\[
y = \frac{1}{2} e^x + \frac{3}{2} e^{-x} - x^2 - 2
\]

Problem 2: Find a particular solution to the DE
\[ y'' - y' - 2y = 3x + 4 \]

Solution: \( y_p = Ax + B \rightarrow 0 - A - 2(Ax + B) = 3x + 4 \rightarrow A = -\frac{3}{2} \), \( B = -\frac{5}{4} \), so \( y_p = -\frac{1}{4} (6x + 5) \)

Problem 3: Find a particular solution to the DE
\[ y^{(3)} + 4y' = 3x - 1 \]
Solution: Since \( y' = 0 \) for \( y \) = constant, try

\[
y_p = Ax^2 + Bx = x(Ax + B)
\]
\[
y_p' = 2Ax + B
\]
\[
y_p'' = 2A
\]
\[
y_p''' = 0
\]

Thus, \( y_p''' + 4y_p' = 4(2Ax + B) = 3x - 1 \rightarrow 8A = 3, 4B = -1 \), so \( A = \frac{3}{8} \) and \( B = -\frac{1}{4} \). Thus,

\[
y_p = \frac{1}{8}(3x^2 - 2x)
\]