18.03SC Unit 3 Exam Solutions

1. (a) The minimal period is 2.
   (b) \( f(t) \) is even.
   (c) \( x_p(t) = \frac{1}{\omega_n^2} + \frac{\cos(\pi t)}{2(\omega_n^2 - \pi^2)} + \frac{\cos(2\pi t)}{4(\omega_n^2 - 4\pi^2)} + \frac{\cos(3\pi t)}{8(\omega_n^2 - 9\pi^2)} + \cdots \)
   (d) There is no periodic solution when \( \omega_n = 0, \pi, 2\pi, 3\pi, \ldots \)

2. (a) 
   ![Graph](image1)
   or 
   ![Graph](image2)

   (b) 
   ![Graph](image3)
   or 
   ![Graph](image4)

(c) \( f'(t) = (u(t+1) - u(t-1)) - \delta(t+1) - \delta(t-1) \).

3. (a) \( v(t) = w(t) * u(t) = \int_0^t w(t-\tau)u(\tau)\,d\tau = \int_0^t (e^{-(t-\tau)} - e^{-3(t-\tau)})\,d\tau \)
   \( = e^{-t} e^{3t} \bigg|_0^t - e^{-3t} \bigg|_0^t = (1 - e^{-t}) - \frac{1 - e^{-3t}}{3} = \frac{2}{3} - e^{-t} + \frac{e^{-3t}}{3} \).
   (b) \( W(s) = \mathcal{L}[w(t)] = \frac{1}{s+1} - \frac{1}{s+3} \).
   (c) \( W(s) = \frac{1}{s+1} - \frac{1}{s+3} = \frac{(s+3) - (s+1)}{(s+1)(s+3)} = \frac{2}{s^2 + 4s + 3} \), so \( p(s) = \frac{1}{2} (s^2 + 4s + 3) \).

4. (a) \( \frac{s-1}{s} = 1 - \frac{1}{s} \rightarrow \delta(t) - u(t), \) so \( \frac{e^{-s}(s-1)}{s} \rightarrow \delta(t-1) - u(t-1) \).
   (b) \( F(s) = \frac{s+10}{s^3+2s^2+10s} = \frac{a}{s} + \frac{b(s+1) + c}{(s+1)^2 + 9} \). By coverup, \( a = \frac{10}{10} = 1 \). By complex coverup (multiply through by \( (s+1)^2 + 9 \) and set \( s \) to be a root, say \(-1 + 3i\)), \( b(3i) + c = \frac{9+3i}{-1+3i} = -3i \), so \( b = -1, c = 0 \), and \( F(s) = \frac{1}{s} - \frac{s+1}{(s+1)^2 + 9} \), which is the Laplace transform of \( 1 - e^{-t} \cos(3t) \).

5. (a) Poles at \( \{0, -1+3i, -1-3i\} \).
   (b) \( X(s) = W(s)F(s). \) \( F(s) = \frac{2}{s^2+4} \), so \( X(s) = \left( \frac{s+10}{s^3+2s^2+10s} \right) \left( \frac{2}{s^2+4} \right) \).
18.03SC Differential Equations
Fall 2011

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