18.03SC Practice Problems 27

Laplace transform

Rules for the Laplace transform

Definition: \[ \mathcal{L}[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)e^{-st} \, dt \quad \text{for} \quad \text{Re}(s) \gg 0. \]

Linearity: \[ \mathcal{L}[af(t) + bg(t)] = aF(s) + bG(s). \]

\( \mathcal{L}^{-1}: \) \[ F(s) \] essentially determines \( f(t) \) for \( t > 0. \)

s-shift rule: \[ \mathcal{L}[e^{rt}f(t)] = F(s - r). \]

s-derivative rule: \[ \mathcal{L}[tf(t)] = -F'(s). \]

t-derivative rule: \[ \mathcal{L}[f'(t)] = sF(s) - f(0^-). \]

Formulas for the Laplace transform

\[ \mathcal{L}[1] = \frac{1}{s}, \quad \mathcal{L}[\delta(t - a)] = e^{-as} \]

\[ \mathcal{L}[e^{rt}] = \frac{1}{s - r}, \quad \mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \]

\[ \mathcal{L}[\cos(\omega t)] = \frac{s}{s^2 + \omega^2}, \quad \mathcal{L}[\sin(\omega t)] = \frac{\omega}{s^2 + \omega^2} \]

1. Use the rules and formulas to find the Laplace transform of \( e^{-t}(t^2 + 1). \)