Part II Problems

Problem 1:  [Periodic solutions] Let \( g(t) \) be the function which is periodic of period \( 2\pi \), and such that \( g(t) = t \) for \(-\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \) and \( g(t) = \pi - t \) for \( \frac{\pi}{2} \leq t \leq \frac{3\pi}{2} \).

(a) Find a periodic solution to \( \ddot{x} + \omega_0^2 x = g(t) \) (if there is one).

(b) For what (positive) values of \( \omega_0 \) are there no periodic solution?

(c) Write \( \omega_r \) for the smallest number you found in (b). For \( \omega_0 \) just less than \( \omega_r \), what is the solution like, approximately? How about for \( \omega_0 \) just larger than \( \omega_r \)?

(d) For what values of \( \omega_0 \) are there more than one periodic solution?

(e) For the values of \( \omega_0 \) found in (d), are all solutions to \( \ddot{x} + \omega_0^2 x = g(t) \) periodic?