Unit Impulse Response: Post-initial Conditions

Quiz: Consider the equation

$$\dot{w} + kw = \delta(t)$$

with rest initial conditions, \(w(0^-) = 0\).

For the solution \(w(t)\) what is \(\dot{w}(0^+)\)?

Choices:

a) \(\dot{w}(0^+) = 0\)

b) \(\dot{w}(0^+) = -1/k\)

c) \(\dot{w}(0^+) = -1\)

d) \(\dot{w}(0^+) = -k\)

e) None of these.

Answer: (d).

Using the DE we get \(\dot{w}(0^+) + kw(0^+) = \delta(0^+)\). We know \(w(0^+) = 1\) and \(\delta(0^+) = 0\). Therefore \(\dot{w}(0^+) = -k\).

We could also look at the solution \(w(t) = e^{-kt}\) for \(t > 0\). Thus \(\dot{w}(t) = -ke^{-kt}\) for \(t > 0\). This implies \(\dot{w}(0^+) = -k\).

Using the solution to the DE probably seems easier than the first method, but it is important to be able to draw conclusions without knowing the solution.