Matrix/Vector Applet

Let us revisit the Matrix/Vector applet.

1. Set the matrix to \[
\begin{pmatrix}
1 & 2 \\
2 & 4 \\
\end{pmatrix}
\]. Can you find a non-zero input vector that produces zero output? What’s the determinant, and why does your finding make sense? Find other matrices where this is the case, and some where it is not.

2. In the previous session, we found some cases where the input vector lined up with the output, and noted that scaling the input did not change this. These are eigenvectors, and the line they span is called an eigenline. Note that if the eigenvalue is negative, the input and output vectors are lined up, but in point in opposite directions.

Can you find:

a) a matrix with exactly two eigenlines;
b) a matrix with exactly one eigenline;
c) a matrix with no eigenlines;
d) a matrix where all lines are eigenlines?

We have already seen some examples, and will see more in the later notes: (a) corresponds to the case of a matrix with two distinct real eigenvalues; (b) to the case of a defective repeated eigenvalue; (c) to the case of complex eigenvalues, and (d) to the case of a complete repeated eigenvalue.