18.04 Exam #3

CLOSED BOOK ... and NO calculators

As before, please struggle with these problems on separate sheets of paper ...

1. Not necessarily via residue calculus, please evaluate

\[ \int_{0}^{\infty} \frac{\ln x}{1 + x^2} \, dx \]

HINTS: Perhaps try ... or just some substitution?

2. a) Which region of the complex z-plane gets mapped by

\[ w = \frac{z - 1}{z + 1} \]

into the interior of the circle \(|w| = 1\), and why?

b) Use the above answer as a clue to find a related bilinear transformation \( W(z) \) that carries the top half of the z-plane into a unit circle centered instead at \( W = 1 + i \).

3. For that "steady-state" solution of \( \frac{dx}{dt} + x = \cos \omega t \)

which can be written as

\[ x(t) = \text{Re} \{ A(\omega) e^{i\omega t} \} \]

show that the amplitude \( A \), regarded as a function of the real "forcing frequency" \( \omega \), traces out a circle in the complex \( A \)-plane.