18.04 Recitation 2  
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1.1. Show directly using the definition of the complex derivative that \( \bar{z} \) is not complex differentiable.

1.2. What are the values that \( \lim_{z \to 0} \frac{\bar{z}}{z} \) can attain?

2.1. What are the real and imaginary parts of \( \cos(z) \)? Of \( \sin(z) \)?

2.2. What are these real and imaginary parts for \( z = x + i0 \)? What about for \( z = 0 + iy \)?

2.3. Is it true that \( \cos(z) \) and \( \sin(z) \) are bounded functions?

2.4. Is it true that \( \cos^2 z + \sin^2 z = 1 \)?

3.1 Show that \( e^z \) is continuous as a function of \( z \).

3.2. Use this to show that \( \cos(z) \) and \( \sin(z) \) are continuous as functions of \( z \).

3.3. Is \( \bar{z} \) continuous as a function of \( z \)? Is this consistent with Problem 1?

4.1. Express the following functions in the form \( f(z) = u(x, y) + iv(x, y) \): \( e^z \), \( z^2 \), \( \cos(z) \) and \( \sin(z) \) (see also Problem 2)

4.2. Compute the partial derivatives \( u_x, u_y, v_x, v_y \) for each of these functions. Do they satisfy the Cauchy-Riemann equations?

4.3. What is \( f'(z) \) in each of these cases?

4.3. Repeat this for \( \bar{z} \). Is this consistent with Problem 1?
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