1.1. Show directly using the definition of the complex derivative that \( \bar{z} \) is not complex differentiable.

**Ans:** Follows from the next part.

1.2. What are the values that \( \lim_{z \to 0} \frac{\bar{z}}{z} \) can attain?

**Ans:** You can get any value on the unit circle. Use the parameterization \( z = re^{i\theta} \), for which the limit becomes \( \lim_{r \to 0} \frac{re^{-i\theta}}{re^{i\theta}} = e^{-2i\theta} \).

2.1. What are the real and imaginary parts of \( \cos(z) \)? Of \( \sin(z) \)?

**Ans:** \( \cos(z) = \cos x \cosh y - i \sin x \sinh y \) and \( \sin(z) = \sin(x) \cosh(y) + i \cos(x) \sinh(y) \).

2.2. What are these real and imaginary parts for \( z = x + i0 \)? What about for \( z = 0 + iy \)?

**Ans:** Follows from above.

2.3. Is it true that \( \cos(z) \) and \( \sin(z) \) are bounded functions?

**Ans:** No. Can see this using the expressions in 2.1.

2.4. Is it true that \( \cos^2 z + \sin^2 z = 1 \)?

**Ans:** Yes. Compute directly using 2.1.

3.1 Show that \( e^z \) is continuous as a function of \( z \).

**Ans:** Write down the real and imaginary parts, and check that each of them is continuous.

3.2. Use this to show that \( \cos(z) \) and \( \sin(z) \) are continuous as functions of \( z \).

**Ans:** Same as above.

3.3. Is \( \bar{z} \) continuous as a function of \( z \)? Is this consistent with Problem 1?

**Ans:** Yes, it is continuous. Yes, it is consistent, since Problem 1 is about differentiability and there are certainly continuous functions which are not differentiable.

4.1. Express the following functions in the form \( f(z) = u(x, y) + iv(x, y) \): \( e^z \), \( z^2 \), \( \cos(z) \) and \( \sin(z) \) (see also Problem 2)

4.2. Compute the partial derivatives \( u_x, u_y, v_x, v_y \) for each of these functions. Do they satisfy the Cauchy-Riemann equations?

4.3. What is \( f'(z) \) in each of these cases?

4.3. Repeat this for \( \bar{z} \). Is this consistent with Problem 1?

**Ans:** See notes for these computations. The computation for \( \cos(z) \) is also on the second problem set. For 4.3, the Cauchy-Riemann equations are not satisfied, which is consistent with what we saw in Problem 1.