1. Let $A = \{ z : |z| \leq 2 \}$, and let $u(x, y)$ be a harmonic function on $A$. Let $B = \{ z : |z| = 2 \}$. Express the following in terms of $u$ and $B$:

1.1. The maximum value of $u$ on $A$.
1.2. The minimum value of $u$ on $A$.
1.3. The value $u(0, 0)$.

2. Let $\Phi(z) = \phi(z) + i\psi(z)$ be an analytic function mapping a region $B$ to another region $A$. Let $u(x, y)$ be a harmonic function on $A$.

2.1. Under the assumption that $A$ is simply connected, show that $u(\phi(x, y), \psi(x, y))$ is a harmonic function on $B$.

2.2. Can we drop the assumption that $A$ is simply connected?

3. Consider the complex potential for the double source: $\Phi(z) = \log(z - 1) + \log(z + 1) = \log(z^2 - 1)$.

3.1. Find the flow $F$.

3.2. Show that on the $y$-axis, the flow is along the axis.

3.3. What are the stagnation points for this flow?

3.4. See the notes for Topic 6 to see the stream lines for this potential and some further discussion.