

## 18.05 Problem Set 7, Spring 2014

**Problem 1.** (10 pts.) **Confident coin: III** (Quote taken from ‘Information Theory, Inference, and Learning Algorithms’ by David J. C. Mackay.)

A statistical statement appeared in *The Guardian* on Friday January 4, 2002:

When spun on edge 250 times, a Belgian one-euro coin came up heads 140 times and tails 110. ‘It looks very suspicious to me’, said Barry Blight, a statistics lecturer at the London School of Economics. ‘If the coin were unbiased the chance of getting a result as extreme as that would be less than 7%’.

- (a) Let  $\theta$  be the probability of coming up heads. Consider the null hypothesis that the coin is fair,  $H_0 : \theta = 0.5$ . Carefully explain how the 7% figure arises. What term describes this value in NHST? Does it correspond to a one-sided or two-sided test?
- (b) Would you reject  $H_0$  at a significance level of  $\alpha = 0.1$ ? What about  $\alpha = 0.05$ ?
- (c) How many heads would you need to have observed out of 250 spins to reject at a significance of  $\alpha = 0.01$ ?
- (d) (i) Fix significance at  $\alpha = 0.05$ . Compute and compare the power of the test for the alternative hypotheses  $H_A : \theta = 0.55$  and  $H'_A : \theta = 0.6$ ?
- (ii) Sketch the pmf of each hypothesis and use it to explain the change in power observed in part (i).
- (e) (i) Again fix  $\alpha = 0.05$ . What is the *smallest* number of spins necessary for the test to have a power of 0.9 when  $H_A : \theta = 0.55$ ?
- (ii) As in part (d), draw sketches and explain how they illustrate the change in power.
- (f) Let  $H_A : \theta = 0.55$ . If we have only two hypotheses:  $H_0$  and  $H_A$  and a flat prior  $P(H_0) = P(H_A) = 0.5$ , what is the posterior probability of  $H_A$  given the data?
- (g) What probability would you personally place on the coin being biased toward heads? Why? There is no one right answer, we are simply interested in your thinking.

**Problem 2.** (10 pts.) **Polygraph analogy.** In a experiment on the accuracy of polygraph tests, 140 people were instructed to tell the truth and 140 people were instructed to lie. Testers use a polygraph to guess whether or not each person is lying. By analogy, let’s say  $H_0$  corresponds to the testee telling the truth and  $H_A$  corresponds to the testee lying.

- (a) Describe the meaning of type I and type II errors in this context, and estimate their probabilities based on the table.

	Testee is truthful	Testee is lying
Tester thinks testee is truthful	131	15
Tester thinks tested is lying	9	125

- (b) In NHST, what relationships exist between the terms significance level, power, type 1 error, and type 2 error?

**Problem 3.** (10 pts.) We perform a  $t$ -test for the null hypothesis  $H_0 : \mu = 10$  at significance level  $\alpha = 0.05$  by means of a dataset consisting of  $n = 16$  elements with sample mean 11 and sample variance 4.

- (a) Should we reject the null hypothesis in favor of  $H_A : \mu \neq 10$ ?
- (b) What if we test against  $H'_A : \mu > 10$ ?

**Problem 4.** (10 pts.) (**z-test**) Suppose three radar guns are set up along a stretch of road to catch people driving over the speed limit of 40 miles per hour. Each radar gun is known to have a normal measurement error modeled on  $N(0, 5^2)$ . For a passing car, let  $\bar{x}$  be the average of the three readings. Our default assumption for a car is that it is not speeding.

- (a) Describe the above story in the context of NHST. Are the most natural null and alternative hypotheses simple or compound?
- (b) The police would like to set a threshold on  $\bar{x}$  for issuing tickets so that no more than 4% of the tickets are given in error.
  - (i) Use the NHST description in part (a) to help determine what threshold should they set.
  - (ii) Sketch a graph illustrating your reasoning in part (i).
- (c) What is the power of this test with the alternative hypothesis that the car is traveling at 45 miles per hour? How many cameras are needed to achieve a power of .9 with  $\alpha = 0.04$ ?

**Problem 5.** (10 pts.) One generates a number  $x$  from a uniform distribution on the interval  $[0, \theta]$ . One decides to test  $H_0 : \theta = 2$  against  $H_A : \theta \neq 2$  by rejecting  $H_0$  if  $x \leq 0.1$  or  $x \geq 1.9$ .

- (a) Compute the probability of a type I error.
- (b) Compute the probability of a type II error if the true value of  $\theta$  is 2.5.

**Problem 6.** (10 pts.) Give a careful, **precise** explanation of the following XKCDs.

- (a) <http://xkcd.com/1132/>
- (b) <http://xkcd.com/882/>

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