Your PRINTED name is: _________________________________ 1.
Your recitation number is ____________________________ 2.
3.

1. (40 points) Suppose $u$ is a unit vector in $\mathbb{R}^n$, so $u^T u = 1$. This problem is about the $n$ by $n$ symmetric matrix $H = I - 2uu^T$.

(a) Show directly that $H^2 = I$. Since $H = H^T$, we now know that $H$ is not only symmetric but also ____________________________.

(b) One eigenvector of $H$ is $u$ itself. Find the corresponding eigenvalue.

(c) If $v$ is any vector perpendicular to $u$, show that $v$ is an eigenvector of $H$ and find the eigenvalue. With all these eigenvectors $v$, that eigenvalue must be repeated how many times? Is $H$ diagonalizable? Why or why not?

(d) Find the diagonal entries $H_{11}$ and $H_{ii}$ in terms of $u_1, \ldots, u_n$. Add up $H_{11} + \ldots + H_{nn}$ and separately add up the eigenvalues of $H$. 
2. (30 points) Suppose $A$ is a positive definite symmetric $n$ by $n$ matrix.

(a) How do you know that $A^{-1}$ is also positive definite? (We know $A^{-1}$ is symmetric. I just had an e-mail from the International Monetary Fund with this question.)

(b) Suppose $Q$ is any orthogonal $n$ by $n$ matrix. How do you know that $Q A Q^T = Q A Q^{-1}$ is positive definite? Write down which test you are using.

(c) Show that the block matrix

$$
B = \begin{bmatrix}
A & A \\
A & A
\end{bmatrix}
$$

is positive semidefinite. How do you know $B$ is not positive definite?
3. **(30 points)** This question is about the matrix

\[
A = \begin{bmatrix}
0 & -1 \\
4 & 0 \\
\end{bmatrix}
\]

(a) Find its eigenvalues and eigenvectors.

Write the vector \( u(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \) as a combination of those eigenvectors.

(b) Solve the equation \( \frac{du}{dt} = Au \) starting with the same vector \( u(0) \) at time \( t = 0 \).

In other words: the solution \( u(t) \) is what combination of the eigenvectors of \( A \)?

(c) Find the 3 matrices in the Singular Value Decomposition \( A = U \Sigma V^T \) in two steps.

– First, compute \( V \) and \( \Sigma \) using the matrix \( A^T A \).

– Second, find the (orthonormal) columns of \( U \).
18.06 Linear Algebra
Spring 2010

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.