18.099 - 18.06CI.

Attention: Next week at MIT Tuesday is a virtual Monday; and Monday is a holiday.
Due on Tuesday, Feb 17 in class. First draft due on Thursday, Feb 12.

(1) Give an example of a linear space over the rationals which is not a linear space over the reals. Can you find an example of a real linear space which is not a rational linear space? Explain your answer.

(2) Let $A$ be the linear space of real infinite sequences $(a_1, a_2, a_3, \ldots)$ over $\mathbb{R}$ with coordinatewise addition and multiplication by numbers. Which of the following are linear subspaces in $A$?
(a) Sequences with only finitely many nonzero terms;
(b) Sequences with only finitely many zero terms;
(c) Cauchy sequences, namely $\{a_i\}_{i=1}^\infty$ satisfying the following condition: for every $\varepsilon$ there exists a number $N > 0$ such that $|a_n - a_m| < \varepsilon$ for all $m, n > N$;
(d) Sequences $\{a_i\}_{i=1}^\infty$ for which the series $\sum_{i=1}^\infty a_i^2$ converges.
Prove your answers.