Exercises on column space and nullspace

Problem 6.1: (3.1 #30. Introduction to Linear Algebra: Strang) Suppose $S$ and $T$ are two subspaces of a vector space $V$.

a) **Definition:** The sum $S + T$ contains all sums $s + t$ of a vector $s$ in $S$ and a vector $t$ in $T$. Show that $S + T$ satisfies the requirements (addition and scalar multiplication) for a vector space.

b) If $S$ and $T$ are lines in $\mathbb{R}^m$, what is the difference between $S + T$ and $S \cup T$? That union contains all vectors from $S$ and $T$ or both. Explain this statement: The span of $S \cup T$ is $S + T$.

Problem 6.2: (3.2 #18.) The plane $x - 3y - z = 12$ is parallel to the plane $x - 3y - x = 0$. One particular point on this plane is $(12, 0, 0)$. All points on the plane have the form (fill in the first components)

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 0 \\ 1 \end{bmatrix}.
\]

Problem 6.3: (3.2 #36.) How is the nullspace $N(C)$ related to the spaces $N(A)$ and $N(B)$, if $C = \begin{bmatrix} A \\ B \end{bmatrix}$?