Exercises on Markov matrices; Fourier series

**Problem 24.1:** (6.4 #7. *Introduction to Linear Algebra*: Strang)

a) Find a symmetric matrix \( \begin{bmatrix} 1 & b \\ b & 1 \end{bmatrix} \) that has a negative eigenvalue.

b) How do you know it must have a negative pivot?

c) How do you know it can’t have two negative eigenvalues?

**Problem 24.2:** (6.4 #23.) Which of these classes of matrices do \( A \) and \( B \) belong to: invertible, orthogonal, projection, permutation, diagonalizable, Markov?

\[
A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad B = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.
\]

Which of these factorizations are possible for \( A \) and \( B \): \( LU \), \( QR \), \( SAS^{-1} \), or \( QΛQ^T \)?
Problem 24.3: (8.3 #11.) Complete $A$ to a Markov matrix and find the steady state eigenvector. When $A$ is a symmetric Markov matrix, why is $x_1 = (1, \ldots, 1)$ its steady state?

\[
A = \begin{bmatrix}
.7 & .1 & .2 \\
.1 & .6 & .3 \\
\_ & \_ & \_
\end{bmatrix}.
\]