Exercises on linear transformations and their matrices

Problem 30.1: Consider the transformation $T$ that doubles the distance between each point and the origin without changing the direction from the origin to the points. In polar coordinates this is described by

$$T(r, \theta) = (2r, \theta).$$

a) Yes or no: is $T$ a linear transformation?

b) Describe $T$ using Cartesian ($xy$) coordinates. Check your work by confirming that the transformation doubles the lengths of vectors.

c) If your answer to (a) was "yes", find the matrix of $T$. If your answer to (a) was "no", explain why the $T$ isn’t linear.

Problem 30.2: Describe a transformation which leaves the zero vector fixed but which is not a linear transformation.