Exercises on positive definite matrices and minima

**Problem 27.1:** (6.5 #33. *Introduction to Linear Algebra: Strang*) When $A$ and $B$ are symmetric positive definite, $AB$ might not even be symmetric, but its eigenvalues are still positive. Start from $ABx = \lambda x$ and take dot products with $Bx$. Then prove $\lambda > 0$.

**Problem 27.2:** Find the quadratic form associated with the matrix \[
\begin{bmatrix}
1 & 5 \\
7 & 9 \\
\end{bmatrix}
\]. Is this function $f(x, y)$ always positive, always negative, or sometimes positive and sometimes negative?