Justify your answers. Cross out what is not meant to be part of your final answer. Total number of points: 50

I. (5 pts) Find all solutions of the equation

\[ z^{4/3} = 1 + i. \]

II. (Total 10 pts)

1. (3 pts) Can the function \( v(x, y) = 4xy + x + y \) be the imaginary part of an analytic function?

2. (5 pts) Determine all the functions \( u(x, y) \) such that \( u(x, y) + iv(x, y) \) is analytic.

3. (2 pts) Find \( f(z) \) such that

\[ f(z) = u(x, y) + iv(x, y). \]

III. (5 pts) Compute the line integral

\[ \int_C \frac{(z^2 - 2)}{z^3} \, dz \]

where \( C \) is the left half-circle joining \(-i\) and \(i\).

IV. (Total 15 pts) Let

\[ f(z) = \frac{z}{(1 - z)(z + 3)}. \]

1. (3 pts) Write \( f(z) \) as a sum of fractions, i.e.,

\[ f(z) = \frac{A}{z - 1} + \frac{B}{z + 3}. \]

2. (2 pts) Find the singularities of \( f(z) \) and classify them;

3. (3 pts) Explain whether it is possible to expand \( f(z) \) in Laurent (or Taylor) power series of:

(i) \( z + 2 \), that converges in the region \( 1 < |z + 2| < 4 \)?

(ii) \( z + 2 \), that converges in the region \( 3 < |z + 2| \)?

(iii) \( z + 2 \), that converges in the region \( 1 < |z + 2| < 3 \)?
4. (7 pts) Write the Laurent series expansion of \( f(z) \) in \(|z - 1| < 2\) as a power series of \((z - 1)\).

V. (Total 8 pts)

1. (5 pts) Let
   \[ f(z) = \frac{1}{(z^2 + z - 2)^2}. \]
   Find the singularities of \( f(z) \), classify them, and compute the residues of \( f(z) \) at those singular points.

2. (3 pts) Compute the integral of \( f(z) \) along the circles of center 0 and radii 1/4, 5/4, and 4, respectively.

VI. (Total 7 pts)

1. (2 pts) Show that the function \( f(z) = \cos z^{1/2} \) is analytic at \( z = 0 \).

2. (5 pts) Show that the function
   \[ f(z) = \frac{\cos z - 1}{\sinh z - z} \]
   has a simple pole at the origin.