1) (30 pts.) This question is about the closest straight line \( b = C +Dt \) to the measurements \( b_1, b_2, b_3 \) at three different times \( t_1, t_2, t_3 \).

What system \( Ax = b \) (especially find \( A \)) would we like to solve for \( C \) and \( D \) but probably can’t? Is \( A^T A \) positive definite or semidefinite?

Suppose the “center of mass” is at \((0,0)\), meaning \( \sum t_i = 0 \) and \( \sum b_i = 0 \). Find the best least squares line \( C + Dt \) (so \( \hat{x} = (C, D) \)). Show that this line goes through the origin \((0,0)\).

What is the condition on \( t_1, t_2, t_3 \) and \( b_1, b_2, b_3 \) for the three points to actually lie on a line (so \( Ax = b \) is solvable without going to least squares)? Various ways to answer, the more specific the better.
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2) (30 pts.) We have a line of two equal masses $m$ connected by three springs with spring constants $c_1 = 1, c_2 = 1, c_3 = S$. Spring 1 is fixed at the top and spring 3 at the bottom, so $x_0 = x_3 = 0$.

Find the stiffness matrix $K$ in the equation $Kx = f$ for the mass displacements. Solve for the displacements $x_1$ and $x_2$.

If the third spring constant $S$ becomes very large or very small ($S \to \infty$ and $S \to 0$) what are the limiting values of the displacements $x$? What are the limiting values of the spring forces $y_1, y_2, y_3$?
3) (40 pts.) The figure shows a network with 8 edges. It is also a plane truss with 8 bars. The lowest two nodes are grounded in the network \((x_6 = x_7 = 0)\) and they are supported in the truss. The upper 5 nodes are free, the edges are numbered.

Find the incidence matrix \(A\) (5 columns) for the network. What are all solutions to \(Ax = 0\)? \textit{How many independent solutions to \(A^T y = 0\) (Current Law)?} (not required to find \(y\)’s).

If \(f_1 = 1\) amp enters node 1 and travels to ground, what equations would you solve for the potentials \(x\) and currents \(y\)? \textit{Assume all} \(c_i = 1\). What are the 8 currents (you can answer from the picture without solving equations)?

What shape is the matrix \(A\) for the truss problem? Describe a complete set of mechanisms (solutions to \(Ax = 0\)). Draw a picture of each mechanism.

For the truss stiffness matrix \(K = A^T A\),

(a) Is \(K\) positive definite? Is it positive semidefinite? Why or why not?

(b) Find a set of horizontal and vertical forces \(f\), not all zero, so that \(Kx = f\) can be solved.