18.085/18.0851 Computational Science Engineering I **Summer 2020**

Homework 3

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3.1 Theoretical Exercises

Prove the following facts

- If $A \in \mathbb{R}^{m\times n}$ has linearly independent columns, then $K = A^T A$ is positive definite
- If $A \in \mathbb{R}^{m\times n}$ has linearly independent columns and C is positive definite, then $K = A^T CA$ is also positive definite.

3.2 SVD by Hand

Perform SVD on the following matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \tag{3.1}$$

3.3 Coding SVD

Code your own image compression script by filling up the needed lines in the SVD_Image.m.
Plot the singular values in sequence. Observe how fast it decays and use your judgment to decide on the best cutoff.

3.4 Normal Equation

Find the best C and D that satisfy the matrix equation below via the least square solution.

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 1 \\ 9 \\ 9 \\ 21 \end{bmatrix} \tag{3.2}$$

3.5 A Calculus Exercise

Let $f = -x^2 - y^2 - z^2 + xy + yz + xz$. We know f has a critical point at $(x, y, z) = 0$. Please characterize this critical point as a local maximum, minimum, or saddle point. Please also comment if you are unable to analytically characterize this critical point.

3.6 Fixed-Free End

In class, we derived the framework for fixed-fixed end. Now imagine that you remove the forth spring and make the system a fixed-free end.

Part I: Derive matrices A and C for the fixed-free end of three masses connected with three springs, assuming Hooke's law $w_i = c_i e_i$, where w_i is the force exerted on spring i, e_i is the elongation of spring i, and c_i is the Hooke's constant.

Part II: Assume $C = I$. Compute the displacement vector u, subject to an external force of $f = (1, 1, 1)^T$