Corrections and Changes to the Third through the Seventh Printings
Revised Oct. 8, 2011

The third printing has 10 9 8 7 6 5 4 3 on the first left-hand page. Later printings end
with higher numbers (currently: 4, 5, 6, or 7).

The list below omits:
minor English typos (doubled periods, wrong punctuation, accidental misspellings);
minor non-confusing mathematical typos: poor spacing is the most common.

Bullets mark the more significant changes or corrections: missing or altered hypotheses,
non-evident typos, new hints or simplifications, etc.

Double bullets mark new exercises or substantially changed ones, or significant changes
to or errors in the text material.

p. 10, Def. 1.6B: read: Any such C . . .

• p. 30, Ex. 2.1/3: replace: change the hypothesis on \{b_n\} by: strengthen the hypotheses
  (cf. p. 405, Example A.1E for the meaning of “stronger”)

• p. 30, Ex. 2.2/1b: read: (make the upper bound sharp)

• 48 Add:
  3-5 Given any \(c\) in \(\mathbb{R}\), prove there is a strictly increasing sequence \(\{a_n\}\) and a strictly
decreasing sequence \(\{b_n\}\), both of which converge to \(c\), and such that all the \(a_n\) and \(b_n\) are
(i) rational numbers; (ii) irrational numbers. (Theorem 2.5 is helpful.)

p. 55, line 7: read: if \(0 < |e_n| < .9\),
p. 57, display (17): read: if \(0 < |e_n| \leq .2\)

• p. 58, Ex. 4.3/2: Omit. (too hard)
• p. 60, Ans. 4.3/2: read: 1024
• p. 63, display (9): delete: \(> 0\)
• p. 63, line 11 from bottom: read: 5.1/4
• p. 68, line 10: replace: hypotheses by: symbols; replace or by and

• p. 69, line 9: read: strictly increasing, clearly \(n_1 \geq 1, n_2 \geq 2\), and so on, so eventually
  lines 11, 13: replace: \(i \geq 1\) by \(i > N\)
• p. 73, line 2: read: \(a_n - L\)
• p. 73, line 6-: read: and estimate it: use 2.4(4), and (16a), suitably applied to \(\{b_n\}\).

• p. 74, Ex. 5.4/1 Add two preliminary warm-up exercises:
  a) Prove the theorem if \(k = 2\), and the two subsequences are the sequence of odd terms
  \(a_{2i+1}\), and the sequence of even terms \(a_{2i}\).
  b) Prove it in general if \(k = 2\).
  c) Prove it for any \(k \geq 2\).

• p. 75, Prob. 5-1(a): replace the first line of the “proof” by:
  Let \(\sqrt[n]{a_n} \to M\). Then by the Product Theorem for limits, \(a_n \to M^2\), so that
p. 82, Proof (line 2): change: \(a_n\ to \(x_n\)
p. 89, Ex. 6.1/1a: change \(c_n\ to \(a_n\)
p. 89, Ex. 6.1/1b add: to the limit \(L\) given in the Nested Intervals Theorem.

• p. 89, Ex. 6.2 add: 3. Find the cluster points of the sequence \(\{\nu(n)\}\) of Problem 5-4.
• p. 90, Ex. 6.3 add: 2. Prove the Bolzano-Weierstrass Theorem without using the
Cluster Point Theorem (show you can pick an $x_{n_i}$ in $[a_i, b_i]$).

p. 90, Ex. 6.5/4: read: non-empty bounded subsets

p. 95, Display (6): delete: $e$

p. 104, l. 10: read: $N + 1$

p. 106, l. 10: read: $\sum(-1)^{n+1}/n$

p. 107, l. 2,3 insert: this follows by Exercise 6.1/1b, or reasoning directly, the picture

p. 108, bottom half through top p.109 replace everywhere: “positive” and “negative” by “non-negative” and “non-positive” respectively

p. 114, line 3- replace: $\leq$ by $<$

p. 115, line 12- read: $|a_n| < 1$

• Question 8.2/2 the series is not Abel-summable; replace by: Show the Abel sum of $0 + 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{4} + \ldots$ is the same as its ordinary sum (cf. 4.2).

p. 121, line 9 read: 8.4A

• p. 122, line 12 replace by: $d_n - e_n$, where $d_n$ and $e_n$ are respectively the two positive series in the line above.

Replace to the end $c^+_n$ and $c^-_n$ by $d_n$ and $e_n$; add after the next paragraph:

Since $d_n$ and $e_n$ are positive series, they are absolutely convergent, and

\[ \sum |c_n| = \sum |d_n - e_n| \leq \sum (|d_n| + |e_n|) = \sum d_n + \sum e_n, \]

which shows that $\sum c_n$ is also absolutely convergent.

• p. 124, Problems add: 8-2 The multiplication theorem for series requires that the two series be absolutely convergent; if this condition is not met, their product may be divergent.

Show that the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ gives an example: it is conditionally convergent, but its product with itself is divergent. (Estimate the size of the odd terms $c_{2n+1}$ in the product.)

• p. 124, 8.2 2. $0 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 - \frac{1}{4}x^4 + \ldots = \ln(1 + x)$; Abel sum is $\ln 2$ (cf. 4.2).

p. 130, line 13: Fourier analysis is devoted to studying to what extent periodic functions

• p. 135, Add hypotheses: $a_1 > 0$, $f(a_1) = 1$, $f(a_2) = 2$.

• p. 143, Example 10.3A and Solution. in $x^4 < x^2$, $x^3 < x^2$ replace $<$ by $\leq$

p. 144, line 4 read: non-zero polynomial

• p. 148, Ex. 10.1/7a(ii) read: is strictly decreasing

p. 154, first line below pictures: read: points of discontinuity

p. 154, line 8 from bottom insert paragraph:

On the other hand, functions like the one in Exercise 11.5/4 which are discontinuous (i.e., not continuous) at every point of some interval are somewhat pathological and not generally useful in applications; in this book we won’t refer to their $x$-values as points of discontinuity since “when everyone is somebody, then no one’s anybody”. If necessary, we will use the oxymoronic “non-isolated point of discontinuity”.

p. 156, line 4 read: In (8) below, the first limit exists if and only if the second and third exist and are equal:

p. 157, line 5 read: $x << -1$

p. 161, line 11: delete ; line 12: read $<$, line 13 read $\leq$

• p. 164, read: Thm.11.4D’ Let $x = g(t)$, $I$ be a $t$-interval, $J$ be an $x$-interval. Then $g(t)$ continuous on $I$, $g(I) \subseteq J$, and $f(x)$ continuous on $J$ $\Rightarrow$ $f(g(t))$ continuous on $I$.

p. 167, Ex. 11.1/4 read: exponential law, $e^{a+b} = e^a e^b$,

• p. 168, Ex. 11.3/3 read: b) $\lim_{x \to 0} \int_0^1 t^2/(1 + t^4) \, dt = 1/3$. 

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p. 168, Ex. 11.3/5 *add*: As \( x \to x_0 \),

- p. 168, Ex. 11.5/2: *rewrite*: Prove \( \lim_{x \to \infty} \sin x \) does not exist by using Theorem 11.5A.
- p. 180, Ex. 12.1/3: *read*: a polynomial

- p. 181, Ex. 12.2/3: *change*: solutions to zeros

p. 183, 12.1/4: *read*: \[ \log_2[(b - a)/e] \]

- p. 192, Ex. 13.1/2 *renumber* as 13.2/2, and *change part (b)* to:

13.2/2b *Is there a continuous function which satisfies the conditions of part (a)? Justify your answer.*

- p. 193, Ex. 13.5/2 *change*: solutions to zeros

- p. 194, Problem 13-7 last two lines, *read*: but for the part of that argument using the compactness of \([a, b]\), substitute part (a) of 13-6 above.

- p. 203, Theorem 14.3B: *label*: Local Extremum Theorem

p. 204, line 4: *read*: an open \( I \)

p. 221, line 11: *read*: \((a; b)\)

Sol’n 15.4/1c: *read*: not one-third!

p. 227, line 2: *read*: \( f'(x) \) not convex

p. 228, Ex. 16.1/1a,b *read*: \([0, 1]\); Ex. 1b: \( x - x^2/2 \)

p. 228, Ex. 16.2/1 *replace*: the second derivative test by each statement in (8)

p. 230, Ans. 16.1/2 *change*: 9 to 0

p. 231, line 3: *change*: \( k \) to \( a \)

- p. 235, display (15): *change*: \( 0 < |c| < |x| \) to \( 0 < c < x, \quad x < c < 0 \).

- p. 243, Example 18.2, Solution, lines 4 and 7 *read*: \([0, x_1]\)

p. 245 lines 1.2: \( f(x_{i-1}) \), line 15: two underscripts: \([\Delta x_i]\)

- p. 248, Ex. 18.2/1 *add*: Hint: cf. Question 18.2/4; use \( x_i^2 - x_{i-1}^2 = (x_i + x_{i-1})(x_i - x_{i-1}) \);

i.e., do it directly, not using the general theorems in 18.3.

- p. 248, Ex. 18.3/1 *replace*: \( n \) by \( k \) everywhere

- p. 260, Defn. 19.6 *read*: \( a = x_0 < x_1 < \ldots < x_{n-1} < x_n = b \)

*add at end*: and has finite left and right limits at each \( x_i \) (just a finite one-sided limit at \( x_0, x_n \)). (Thus \( f(x) \) can have discontinuities only at the \( x_i \), and they are jump or removable discontinuities.)

- p. 261, Solution. a) \( \tan \) \( x \) is piecewise monotone with respect to \( 0, \pi/2, 3\pi/2, 2\pi \), but not piecewise continuous since its limits at \( \pi/2 \) and \( 3\pi/2 \) are not finite.

\[ \text{(b) read: } \frac{1}{n+1}\pi, \frac{1}{n\pi} \]

- p. 261, Lemma 19.6 *rename*: Endpoint Lemma

p. 261, line 7: *replace*: \([c, d]\) by \([a, b]\)

p. 265, Ex. 19.6/1b line 2 *replace*: \( f(x) \) by \( p(x) \)

p. 273, line 2: *read*: (cf. p. 271)

p. 282, line 2: *read*: by interpreting the integral and limit geometrically

p. 289, Ans. 20.5/1: *read*: 1024

p. 291, Ex. 21.1B - line 3: *read*: \( \lim_{R \to \infty} \int_R^0 \)

- line 2: *read*: for \( p > 1 \)

p. 307, Example 22.1C *read*: Show: as \( n \to \infty \), \[ \frac{n}{1 + nx} \ldots \]

p. 310, Theorem 22.B *read*: \( \sum_0^n M_k \)

p. 316, Theorem 22.5A: *delete*: for all \( n \geq 0 \)

- p. 322, Ex. 22.1/3 *read*: \( u_k(x) = \)
p. 332, middle delete both \( \mathbb{N}_1 \), replace the third display by: \( \mathbb{N}_0 = N(\mathbb{Z}) < N(S) < N(\mathbb{R}) \)  

p. 335, lines 6-,7- read: bounded and have only a finite number of jump discontinuities  

• p. 340, delete: last 9 lines of text before Questions 23.4  

p. 350, line 10- read: Subsequence Theorem 5.4  

p. 351, line 5- read: infinite quarter-planes containing the \( x \)-axis and lying between ...  

• p. 353, line 4- read: 24.4A:  
  line 2- read: \( x + y = 2 \)  

p. 354, Theorem 24.5B: read: for all \( x_n \)  

line 7- read: \( f(x_n) \)  

p. 357, Theorem 24.7B, line 2 read: non-empty compact set \( S \):  

line 6 read: bounded and non-empty:  

p. 367, line 15- add: Or make up a simple direct proof.  

p. 369, Theorem 25.3A: (i) read: then \( S = \); (ii) read: \( S = \bigcup U_i \)  

• p. 385, line 2- read: \( \int_0^1 \)  

• p. 388, footnote replace by: We prove the first inequality in (7), which is the analog – for absolutely convergent improper integrals – of the infinite triangle inequality for sums.  

For a fixed \( x \), we have by the Absolute Value Theorem for integrals (19.4C)  

\[
\left| \int_R^S f(x, t) \, dt \right| \leq \int_R^S |f(x, t)| \, dt, \quad \text{for all } S > R, R \text{ fixed.}
\]

As \( S \to \infty \), the right side has the limit \( \int_R^\infty |f(x, t)| \, dt \), since the integral \( \int_R^\infty f(x, t) \, dt \) is assumed to be absolutely convergent.  

The left side has the limit \( |\int_R^\infty f(x, t) \, dt| \), since the integral is convergent (by theorem 21.4), and \( \| \cdot \| \) is a continuous function.  

Finally, by the Limit Location Theorem 11.3C (21), the inequality is preserved as \( S \to \infty \).  

p. 399, line 18- read: \( a(b + c) = ab + ac \)  

p. 404, Example A.1C(i): read: \( a^2 + b^2 = c^2 \)  

p. 415, Ex. A.4/6 read: Fermat’s Little Theorem is the basis of the RSA encryption algorithm, widely used to guarantee website security.  

p. 417, A.4/1 line 1 read: both sides are 1  

A.4/2 line 1: read: \( 2^n + 1 \)  

• p. 429 last 5 lines: replace sentences by:  
  As the picture shows, since \( |f'(x)| > 1.2 \) on \([.7, 1]\), we will have its reciprocal \( |g'(x)| < 1/1.2 \approx .8 \) on the interval \([0, f(.7)] = [0, .83]\).  

This shows Pic-2 is satisfied for \( g(x) \) on the interval \([0, .83] \); the picture shows the root of \( x = g(x) \) will lie in this interval. Thus the Picard method is applicable to \( x = g(x) \). Starting with say \( .7 \), it leads to a root \( \approx .76 \).  

p. 436, Remarks, first paragraph replace \( x^3 \) by \( x^4 \)  

p. 439, top half: \( \text{change } p \text{ and } q \text{ to } P \text{ and } Q \) (to avoid confusion with the use of the real number \( p \) in Example D.4)  

p. 442, line 2 read: \( \geq \)  

line 6 read: \( \leq \)  

• p. 443, Ex. D.2/4: read: Find, by calculating the derivatives for \( x \neq 0 \) and using undetermined coefficients, a second-order linear homogeneous D.E. satisfied by \( y = x^4 \sin(1/x), y(0) = 0, \ldots \)  

p. 459, ruler function: read: 169