Let $E$ and $F$ be two compact subsets of the real numbers $\mathbb{R}$ with the standard (Euclidian) metric $d(x, y) = |x - y|$. Show that the Cartesian product

$$E \times F = \{(x, y) \mid x \in E \text{ and } y \in F\}$$

is a compact subset of $\mathbb{R}^2$ with the metric $d_2(\bar{u}, \bar{v}) = \|\bar{u} - \bar{v}\|_2$.
(Recall that the norm $\| \cdot \|_2$ is defined by $\|(x, y)\|_2 = (x^2 + y^2)^{1/2}$.)

2. Problem # 12 page 44 in Rudin.

3. Problem # 14 page 44 in Rudin.

4. Problem # 16 page 44 in Rudin.

5. Problem # 30 page 46 in Rudin.

6. (a) Show that, for any $\epsilon > 0$, there is a union of intervals with total length $< \epsilon$ that contains the Cantor set $C = \bigcap_{n \in \mathbb{N}} E_n$ (defined in Rudin 2.44). [Hint: $C \subset E_n$, and each of the $2^n$ intervals in $E_n$ is contained in an open interval of length $(1 + \epsilon)/3^n$.]
(b) Show that the Cantor set $C \subset \mathbb{R}$ is compact.

(not for credit) Show that the Cantor set is uncountable – either by fixing the proof of Rudin 2.43, or by using another (e.g. diagonal) argument.