Problem Set 2

1. Consider the subset of \( \mathbb{R} \) defined by

\[
\mathbb{Q}(\sqrt{2}) = \{ a + \sqrt{2}b : a, b \in \mathbb{Q} \},
\]

with the usual addition and multiplication. Show that this is a field (you may use all properties of the real numbers). (2 points)

2. Problem 11 from page 23. (2 points)

3. Problem 9 from page 43, parts (a), (b), (c) and (e) only.  
   *When writing the answer for this problem, please pay particular attention to completeness of the argument; and to structure, clarity and legibility of writing. Typesetting (LaTeX or similar) is encouraged, but not (yet) required. Your answer will be assessed by the grader for correctness, and then again by the recitation instructor for quality of exposition.*  
   (1+1+1+2 = 5 points)

4. Problem 29 from page 45. (4 points)

Total: \( 2 + 2 + 5 + 4 = 13 \) points.