Problem Set 6

1. Given nonzero real numbers $x_1, x_2, \ldots$ define a notion of convergence for the infinite product $\prod_{k=1}^{\infty} x_k$. Prove that if the product converges (in your sense) and the limit is nonzero, then necessarily $\lim_{k} x_k = 1$. On the other hand, prove that $\prod_{k=1}^{\infty} (1 + 1/k)$ does not converge. (1+2+2 = 5 points)

2. Prove that $\sum_{k=1}^{\infty} \frac{1}{k^2} < 7/4$. (3 points)

3. Problem 2 on page 98 (3 points). Please write this problem up carefully in LaTeX.

4. Problem 3 on page 98 (2 points).

Total: $5 + 3 + 3 + 2 = 13$ points.