Example 13.1. The map $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x, y) = xy$, is continuous.

Example 13.2. The map $\exp : \mathbb{R} \to \mathbb{R}$ is continuous (the same holds for $\exp$ of a complex number, hence also for $\cos$ and $\sin$).

Theorem 13.3. (Intermediate Value Theorem) Let $f : [a, b] \to \mathbb{R}$ be a continuous map, such that $f(a) \leq 0$, $f(b) \geq 0$. Then there is some $x$ such that $f(x) = 0$.

Corollary 13.4. Let $f : [a, b] \to \mathbb{R}$ be a continuous map. Then its image is a closed interval $[c, d]$.

Corollary 13.5. Let $f : [a, b] \to \mathbb{R}$ be a continuous map which is strictly increasing ($x < y$ implies $f(x) < f(y)$). Then $f$ is one-to-one onto a closed interval $[c, d]$. Moreover, the inverse map $f^{-1} : [c, d] \to [a, b]$ is continuous.

Example 13.6. The map $\exp$, from the real numbers to the positive real numbers, is strictly increasing and onto. We call its inverse the natural logarithm $\log : (0, \infty) \to \mathbb{R}$. This automatically satisfies $\log(ab) = \log(a) + \log(b)$, and is continuous.

Let $(X, d_X)$ and $(Y, d_Y)$ be metric spaces, and $f : X \to Y$ a map.

Definition 13.7. $f$ is uniformly continuous if: for any $\epsilon > 0$ there is a $\delta > 0$ such that if $d_X(x, y) < \delta$, then $d_Y(f(x), f(y)) < \epsilon$.

Every absolutely continuous map is continuous.

Theorem 13.8. If $X$ is compact, every continuous map $f : X \to Y$ is uniformly continuous.
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