Corollary 3.1. For every real number $x > 0$ there is a natural number $n$ such that $\frac{1}{n} < x$.

Corollary 3.2. For every real number $x$ there is an integer $n$ such that $x < n \leq x + 1$.

Corollary 3.3. For any real numbers $x < y$ there is a rational number $q$ such that $x < q < y$.

Definition of decimal expansion

$$0.9999\ldots \sup\{0, 0.9, 0.99, 0.999, \ldots\}$$

Theorem 3.4. $0.9999\ldots = 1$.

Theorem 3.5. Let $I_1 \supset I_2 \supset I_3 \cdots$ be nonempty closed intervals, $I_k = [a_k, b_k]$.
Then

$$\bigcap_{k=1}^{\infty} I_k \neq \emptyset.$$ 

Corollary 3.6. $\mathbb{R}$ is uncountable.

Definition of complex numbers and their usual operations.

Theorem 3.7. (Cauchy-Schwarz) For complex numbers $z_1, \ldots, z_k, w_1, \ldots, w_k$,

$$|z_1\bar{w}_1 + \cdots + z_k\bar{w}_k|^2 \leq (|z_1|^2 + \cdots + |z_k|^2)(|w_1|^2 + \cdots + |w_k|^2).$$