18.100C Lecture 7 Summary

**Theorem 7.1.** Let \((X,d)\) be a metric space with the following property: every countably infinite subset \(E \subset X\) has a limit point. Then \(X\) is compact.

Step 1: show that \(X\) has an at most countable dense subset (homework).
Step 2: show that if \((U_i)_{i \in I}\) is an open cover of \(X\), then at most countably many \(U_i\) already cover \(X\).
Step 3: show that if \((U_i)_{i \in I}\) is a countable open cover of \(X\), then finitely many \(U_i\) already cover \(X\).

**Theorem 7.2** (Heine-Borel). Every finite closed interval \([a,b] \subset \mathbb{R}\) is compact (for the standard metric).

**Theorem 7.3.** Every bounded closed subset of \(\mathbb{R}\) is compact.

**Theorem 7.4.** Every finite closed cube \([a_1,b_1] \times \cdots \times [a_n,b_n] \subset \mathbb{R}^n\) is compact.

**Theorem 7.5.** Every bounded closed subset of \(\mathbb{R}^n\) is compact.