TOPICS FOR THE FIRST TEST FOR 18.102, SPRING 2009
TEST: THURSDAY 5 MARCH, 9:30-11:00.

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Completeness of answers will be taken strong into account on the test. You are supposed to get things really right!

1. QUESTION 1

Will be on proving some property of integrable functions directly from the definition. Thus, just assuming that $f$ is integrable (you will be asked to give the definition accurately) show that $|f|$, Re $f$, Im $f$, (Re $f$)$_+$ (the positive part) or $f + g$ for two integrable functions, or max($f, g$) for two real integrable functions or some such.

2. QUESTION 2

Monotonicity for step functions. I will ask you to prove one of the two basic monotonicity results for step functions – e.g. that if $f_n$ is an increasing sequence with non-negative pointwise limit then $\lim f_n \geq 0$ (including possibly $+\infty$.) Or the result on which this depends.

3. QUESTION 3

I suggested I might ask you to prove the continuity-in-the-mean of $L^1$ functions. That is to show that

$$\lim_{t \to 0} \int |f_t - f| = 0$$

for any $f \in L^1$.

4. QUESTION 4

There probably will be no question 4, if there is, it would be about null functions.

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