18.175: Lecture 20

Infinite divisibility and Lévy processes

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Outline

Infinite divisibility

Higher dimensional CFs and CLTs
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Higher dimensional CFs and CLTs
Say a random variable $X$ is **infinitely divisible**, for each $n$, there is a random variable $Y$ such that $X$ has the same law as the sum of $n$ i.i.d. copies of $Y$.

What random variables are infinitely divisible?

Poisson, Cauchy, normal, stable, etc.

Let’s look at the characteristic functions of these objects. What about compound Poisson random variables (linear combinations of independent Poisson random variables)? What are their characteristic functions like?

What if have a random variable $X$ and then we choose a Poisson random variable $N$ and add up $N$ independent copies of $X$.

More general constructions are possible via Lévy Khintchine representation.
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Higher dimensional CFs and CLTs
Much of the CLT story generalizes to higher dimensional random variables.

For example, given a random vector \((X, Y, Z)\), we can define
\[
\phi(a, b, c) = E e^{i(aX + bY + cZ)}.
\]

This is just a higher dimensional Fourier transform of the density function.

The inversion theorems and continuity theorems that apply here are essentially the same as in the one-dimensional case.