TOPICS: First order scalar pde.
    Examples of solutions by characteristics.
    Domain of influence.

Review characteristics.
Examples in detail:
1) \(x \cdot u_x + y \cdot u_y = 0\),
   for \(y \geq 1\), with \(u(x, 1) = F(x)\)
2) \(x \cdot u_x + y \cdot u_y = 1+y^2\),
   for \(y \geq 1\), with \(u(x, 1) = F(x)\)

Domain of dependence and domain of influence. Where is the solution defined and where it is not.

Examples showing solution not unique outside domain of influence:

For case (1), with \(F(x) = \exp(-x^2)\), consider (in the plane without the origin = P0)

\[u_1 = \exp(-x^2/y^2)\] .......... for \(x^2+y^2 > 0\).

\[u = \exp(-x^2/y^2)\] .......... for \(y \geq 0\) and \(x^2+y^2 > 0\).
\[= \exp(-3x^2/y^2)\] .......... for \(y \leq 0\) and \(x^2+y^2 > 0\).

Both \(u_1\) and \(u_2\) are smooth and solve the equation and given data, but they are not equal outside \(y \geq 0\) and \(x^2+y^2 > 0\). Can construct infinitely many such \(u's\).