18.310 IMPROVING INFORMATION ORDER AND CONNECTIVITY

1. Revise the sentence to better fit each context.

Familiar information — **Important new information**

Enumeration of **binary trees** was achieved by defining a **bijection** between binary trees and Dyck paths.

Important new information — **Familiar information**

Enumeration of binary trees was achieved by using a **bijection between binary trees and Dyck paths**.

2. Revise to improve information order and connectivity. This text is from a section about counting. Catalan numbers are a familiar concept. The new concept here is binary trees.

A **binary tree** is a plane tree in which vertices have either 0 or 2 children. Binary trees can be counted by the Catalan sequence.

3. Read the theorem and proof below. Starting with the second sentence of the proof, identify the familiar information and the important new information in each sentence. Then revise to improve the information order and connectivity, without revising the theorem statement.

Theorem. *For each n, the set $\mathcal{T}_n$ of plane trees with n edges is the same size as the set $\mathcal{D}_n$ of Dyke paths with $2n$ steps:*

$$|\mathcal{T}_n| = |\mathcal{D}_n| \text{ for all } n.$$

Proof. We define a bijection Φ between plane trees and Dyck paths as follows: given any tree $T \in \mathcal{T}_n$, perform a *depth-first search* of the tree T (as illustrated in Figure 0.1) and define $\Phi(T)$ as the sequence of up and down steps performed during the search. A Dyke path $D \in \mathcal{D}_n$ is obtained from T because $\Phi(T)$ has n up steps and n down steps (one step in each direction for each edge of T), starts and ends at level 0, and remains non-negative. Because Φ is a bijection between $\mathcal{T}_n$ and $\mathcal{D}_n$, we conclude $|\mathcal{T}_n| = |\mathcal{D}_n|$. □

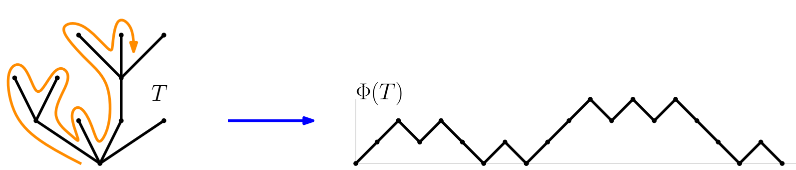

FIGURE 0.1. A plane tree T and the associated Dyke path $\Phi(T)$. The depth-first search of the tree is represented graphically by a tour around the tree (drawn in orange): first visit the leftmost subtree entirely, then the next subtree, etc.

For more information about connectivity, see "The Science of Scientific Writing," by Gopen & Swan. Published in *American Scientist* 78(6) 550-558, Nov-Dec 1990 and available online.

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