Here we are only interested in real matrices, though the results have easy generalizations to complex matrices.

a) A symmetric matrix has real eigenvalues, and the eigenvectors can be taken to be an orthonormal basis.

b) If \( \{n_j\} \), \( 1 \leq j \leq N \), is an orthonormal basis of \( \mathbb{R}^N \), then any vector \( Y \) can be written in the form
\[
Y = \sum_{1}^{N} y_j n_j,
\]
where
\[
y_j = \langle n_j, Y \rangle
\]
\( \langle \cdot, \cdot \rangle = \) scalar product.

c) Let \( A \) be a real-diagonalizable square matrix. Let \( R_j \), \( 1 \leq j \leq N \), be a set of \( N \) linearly independent right (column) eigenvectors of \( A \)
\[
A R_j = \lambda_j R_j,
\]
where the lambda’s are the eigenvalues [they need not be distinct]. Then a set of \( N \) linearly left (row) eigenvectors of \( A \)
\[
L_j A = \lambda_j L_j
\]
can be selected such that
\[
L_n^T R_m = \delta_{\{n, m\}},
\]
where \( \delta_{\{n, m\}} \) is the Kronecker delta. Then any (column) vector \( Y \) can be written in the form
\[
Y = \sum_{1}^{N} y_j R_j,
\]
where
\[
y_j = L_j^T Y.
\]

This formula generalizes the one in (a-b), which applies for symmetric matrices.