Show equivalent to wave equation:
Eliminate either $R$ or $u$. Better yet, introduce velocity potential:

$$u = \phi_x\quad \text{and} \quad (a_0^2/\rho_0) R = - \phi_t$$

Note this is the pressure perturbation.

Hence, second equation is satisfied. Then first equation gives

$$\phi_{tt} - a_0^2 \phi_{xx} = 0. \quad \text{Wave equation.}$$

Note boundary conditions:
- Closed pipe: $\phi_x = 0$.
- Open pipe: $\phi = 0$.

Give other examples where these boundary conditions occur:
- Shallow water: Closed and open channel.
- String equation: Free end and clamped.

Using the solution above for gas-dynamics, we see that

$$\phi = F(x - a_0 t) + G(x + a_0 t),$$

where $F' = (1/2) f$ and $G' = (1/2) g$. 
